MATH 1401 SPRING 2000 CHEAT SHEET FINAL

JAN MANDEL

1. Important formulas from algebra. $\sin(a+b)=\sin(a)\cos(b)+\cos(a)\sin(b)$, $\sin^2 x+\cos^2 x=1$, $a^{b+c}=a^b a^c$, $a^{m/n}=\sqrt[n]{a^m}$, $a^b=e^{(\log a)b}$. Solution of $ax^2+bx+c=0$ is $x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

2. Limits and continuity. $\lim_{x\to c} f(x)=f(c) \iff f$ is continuous at c
$\lim_{x\to 0}\frac{\sin(x)}{x}=1$, $\lim_{x\to 0}\frac{1-cos(x)}{x}=0, \lim_{x\to 0}(1+x)^{1/x}=e$
$\lim_{x\to c} f(x)=L \iff \lim_{x\to c^-} f(x)=\lim_{x\to c^+} f(x)=L$
Intermediate value theorem: If f is continuous on [a,b] and k is between $f(a)$ and $f(b)$, then there exists $c\in[a,b]$ such that $f(c)=k$.
Infinite limits: The formulas for the limit of sum, product, and quotient apply unless they lead to undefined expressions of the form $\infty-\infty$, $\infty.0$, $L/0$, ∞/∞.
If $\lim_{x\to c} f(x)\neq 0$ and $\lim_{x\to c} g(x)=0$, with $g(x)\neq 0$ on a neighborhood of c, then the graph of f/g has vertical asymptote $x=c$.

3. Differentiation. The equation of the line passing through (x_0,y_0) with slope s is $y-y_0=s(x-x_0)$. The equation of the tangent to the graph of f at (x_0,y_0), $y_0=f(x_0)$, is $y-y_0=f'(x_0)(x-x_0)$.
$f'(c)=\lim_{x\to c}(f(x)-f(c))/(x-c)$. If $f'(c)$ exists, f is continuous at c.
$(x^n)'=nx^{n-1}$, $(\sin x)'=\cos x$, $(\cos x)'=-\sin x$, $(\ln x)'=1/x$, $(e^x)'=e^x$
$\sin' x=\cos x$, $\cos' x=-\sin x$, $(\arctan x)'=1/(1+x^2)$, $\arcsin' x=\frac{1}{\sqrt{1-x^2}}$, $\text{arcsec}' x=\frac{1}{|x|\sqrt{1-x^2}}$, $(uv)'=u'v+uv'$, $(u/v)'=(u'v-uv')/v^2$, $f(g(x))=f'(g(x))g'(x)$
If $g=f^{-1}$ and $y=g(x)$, $f'(y)\neq 0$, then $g'(x)=1/f'(y)$.

4. Applications and extrema. If f is continuous on $[a,b]$, then f attains maximum and minimum on $[a,b]$. f can attain extremum on $[a,b]$ only at endpoints or critical numbers (where f' does not exist or $f'=0$). f can attain relative extremum in (a,b) only at a critical number.
Mean value theorem: If f is continuous on $[a,b]$ and differentiable on (a,b) then there exists $c\in(a,b)$ such that $f'(c)=(f(b)-f(a))/(b-a)$. (The case when $f(a)=f(b)$ is Rolle's theorem.)
If $f'>0$ in (a,b) and f is continuous on $[a,b]$, then f is increasing on $[a,b]$.
If f is continuous at c, $f'(x)<0$ for $x<c$ and $f'(x)>0$ for $x>c$, then f has relative minimum $(c,f(c))$. (Or, relative minimum $f(c)$ at $x=c$.)
If f' in increasing in interval I, then f is concave upward in I.
If $f''>0$ in (a,b), then f is concave upward in (a,b).
If $f'(c)=0$ and $f''(c)>0$, then f has relative minimum at c.

5. Hyperbolic functions. $\sinh x=\frac{e^x-e^{-x}}{2}$, $\cosh x=\frac{e^x+e^{-x}}{2}$, $\cosh^2 x-\sinh^2 x=1$, $\cosh' x=\sinh x$, $(\tanh^{-1})'=1/(1-x^2)$

6. Integration. $\int f(x)\,dx=F(x)+C$, $F'=f$.
$\int x^n\,dx=x^{n+1}/(n+1)+C, n\neq -1$, $\int f(g(x))g'(x)\,dx=\int f(u)\,du$, $u=g(x)$
$\int\frac{1}{\sqrt{a^2-x^2}}=\arcsin\frac{x}{a}+C$, $\int\frac{1}{a^2+x^2}\,dx=\frac{1}{a}\arctan\frac{x}{a}+C$
$\int\frac{1}{\sqrt{a^2+x^2}}=\sinh^{-1}\frac{x}{a}+C$, $\int\frac{1}{a^2-x^2}\,dx=\frac{1}{a}\tanh^{-1}\frac{x}{a}+C$
$\int_a^b f(x)\,dx=F(b)-F(a)$, $F'=f$.
$(d/dx)\int_a^x f(t)\,dt=f(x)$